Integral University, Lucknow
Department of Mathematics

Pre Ph. D. course work

Subject: NUMERICAL METHODS **Subject Code: MT601**

(w.e.f. session 2015-2016)

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Unit I

Algebraic and Transcendental Equations: Bisection method, Iteration method, False position method, Newton-Raphson method, Rate of convergence of methods, Solution of system of non-linear equations by Newton Raphson's method.

Unit II

Solution of Simultaneous Equation: Gauss-Seidel method, Gauss-elimination method, Jacobi method.

Iterative Method for Eigen Values: Power method, Jaccobi method.

Unit III

Interpolation: Difference operators, difference tables, factorial polynomials, Newton's formula, Gauss, Stirling, Bessel's formulae for equal interval, Lagrange's formula and Newton's divided difference formula for unequal intervals.

Unit IV

Numerical Integration and Differentiation: Introduction, numerical differentiation, numerical integration by Trapezoidal rule, Simpson's 1/3 rule, Simpson's 3/8 rule, Euler-Maclaurin's formula, Double integration using Trapezoidal and Simpson's rule.

Unit V

Solution of Ordinary Differential Equations: Taylor's series method, Euler's method, Runge-Kutta method of fourth order, Picard's method for successive approximation, Finite difference method, Taylor series method for second order differential equations.

Reference Books:

I . Sastry : Introductory method of numerical analysis, PHI.

2. Balaguruswamy : Numerical method, TMH.

3. A. Ralston and P. Rabinonowitz : First course in numerical analysis, Mc. Graw Hills (ISE).

4. Jain, lyengar, Jain : Numerical method for scientific & engineering computations, New Age International.

5. P. Kandaswamy : Numerical method, S. Chand & company.

Integral University, Lucknow
Department of Mathematics

Pre Ph. D. course work

Subject: Geometry of Differentiable Manifolds **Subject Code: MT602**
(w.e.f. session 2015-2016)

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3	1	0

Unit I
Almost contact Structures, Properties of Almost contact structures, Contact metric structures.

Unit II
Almost complex structures, some basic properties of almost complex structures.

Unit III
Submanifolds, Gauss, Weingarten equations and Curvatures tensors.

Unit IV
CR-submanifolds of Sasakian and Kaehlerian manifolds.

Unit V
Semi-invariant submanifolds of Sasakian and Kaehlerian manifolds.

Reference Books:

1. Differentiable structures and their applications
   By Prof. R.S Mishra, Chandrama Prakashan, Allahabad

2. Contact Manifolds in Riemannian Geometry
   By David E. Blair, Springer-Verlag

3. Geometry of submanifolds
   By B.Y. Chen, Marcel Dekker, New York.

4. Geometry of CR- submanifolds
   By Aurel Bejancu, D.Reidel Publishing Co.Holland

Integral University, Lucknow
Department of Mathematics

Pre Ph. D. course work

Subject: Optimization Theory **Subject Code: MT603**

(w.e.f. session 2015-2016)

L	T	P
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Unit I

Linear Programming: Linear programming- modeling and examples, Simplex algorithm-its theory and computational procedure. Artificial variable technique: Big `M' method and two phase method, Duality in linear programming.

Unit II

Integer Programming: Formulation of various industrial problems as integer and mixed integer programming Problems, Branch and bound algorithm, Cutting plane methods for pure and mixed integer programming problems.

Unit III

Project Management (CPM and PERT Analysis): Introduction to network analysis, Definition of a project, job and events, Drawing of network diagrams, Determination of critical paths and calculation of floats.

Unit IV

Game Theory: Two person zero sum games, Pure and mixed strategies, Maximin and minimax principle, Principle of dominance, Solution of game by graphical method.

Unit V

Combinatorics: Fundamental principles, Permutation and combination, Binomial theorem, recurrence relation, Generating function.

Reference Books:

1. D.F. Stanat & D.E. McAllister: "Discrete Mathematics in Computer Science", Prentice Hall
2. D.G. Luenberger: "Introduction to Linear and Non Linear Programming", AddisonWesley Publishing Co.
3. F. Hiller & G. Liebermann: "Introduction to Mathematical Programming", McGraw Hill
4. Mokhtar, S. Bazara & C.M. Shetty: "Non Linear Programming", J. Wiley

Integral University, Lucknow

Department of Mathematics

Pre Ph. D. course work

Subject: Functional Analysis **Subject Code: MT604**

(w.e.f. session 2015-2016)

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UNIT-I

vector space, normed space, Banach space, incomplete normed space, convex set, properties of normed space, subspace, completeness and compactness of normed space, finite dimensional normed space, continuous and bounded linear operators, dual space, normed spaces of the operators.

UNIT-II

Inner product space, Hilbert space, orthogonality, properties of Hilbert space, orthonormal sets and total orthonormal sets, Schwarz inequality, orthogonal complements and direct sums.

UNIT-III

Functional on Hilbert spaces, Riesz's Representation Theorem, Fundamental theorems for Normed and Banach spaces: Uniform boundedness Theorem, Open Mapping Theorem, close graph theorem.

UNIT-IV

Zorn's lemma, Hahn-Banach Theorem, Hahn-Banach Theorem for Complex Vector spaces and Normed Spaces, Baire's Category Theorem, Banach fixed point theorem.

UNIT-V

Reflexive spaces, Bounded linear functionals, Riesz-Frechet theorem, Adjoint operator, Hilbert-adjoint operators, self adjoint operators, normal operators and unitary operators.

REFERENCES:

1. *Introductory Functional Analysis with Applications* by Erwin Kreyszig, (1989).
2. *Introduction to functional analysis with applications* by A.H.Siddiqi, Khalil Ahmad and P.Manchanda , Real World Education Publishers, New Delhi (2015).
3. *Applied Functional Analysis* by A.H.Siddiqi, Real World Education Publishers, New Delhi (2015).
4. *Elements of the Theory of Functions and Functional Analysis* by W. *Rudin*.

Integral University, Lucknow

Department of Mathematics

Pre Ph. D. course work

Subject: Special Functions **Subject Code: MT605**

(w.e.f. session 2015-2016)

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Unit-I HYPERGEOMETRIC FUNCTIONS OF SEVERAL VARIABLES

Definition of hypergeometric functions, Properties of the hypergeometric function, confluent hypergeometric functions, Properties of the confluent hypergeometric function, Whipple's theorem, Dixon's theorem, Definition and properties of the hypergeometric functions of several variables, Generating functions and series arrangement technique, Examples, Problems.

Unit-II LEGENDRE POLYNOMIALS AND FUNCTIONS

Legendre's equation and its solution, Generating function for the Legendre polynomials, further expressions for the Legendre polynomials, explicit expressions for and special values of the Legendre Polynomials, Orthogonality properties of the Legendre polynomials, Legendre series, Relations between the Legendre polynomials and their derivatives, recurrence relations, Examples, Problems.

Unit-III BESSEL FUNCTIONS

Bessel's equation and its solutions, Bessel function of the first and second kind, Generating function for the Bessel functions, Integral representations for Bessel functions, Recurrence relations, Equations reducible to Bessel's equation, Modified Bessel functions, Recurrence relations for the modified Bessel functions, Integral representations for the modified Bessel functions, Orthonormality of the Bessel functions; Bessel series, Integrals involving Bessel functions, Examples, Problems.

Unit-IV HERMITE POLYNOMIALS

Hermite's equation and its solution, Generating function, Other expressions for the Hermite polynomials, Explicit expressions for and special values of the Hermite polynomials, Orthogonality properties of the Hermite polynomials, Relations between Hermite polynomials and their derivatives, recurrence relations, Examples, Problems.

Unit-V LAGUERRE POLYNOMIALS

Laguerre's equation and its solution, Generating function, Alternative expression for the Laguerre polynomials, Explicit expressions for and special values of the Laguerre polynomials, Orthogonality properties of the Laguerre polynomials, Relations between Laguerre polynomials and their derivatives, recurrence relations, Examples, Problems.

Reference Books:

1. Special Functions for Scientists and Engineers, W.W. Bell, D. Van Nodtrand Company Ltd., London, 1968.
2. Special Function, E.D.Rainville, The Macmillan Company, New York, 1960.
3. A Treatise on Generating Function, H.M. Srivastava and H.L. Manocha, Ellis Horwood Limited, New York, 1984.
4. Special Functions and Calculus of Variations, M.A. Pathan, P.K. Benarji, V.B.L. Chaurasia, M.C. Goyal, Ramesh Book Depot., Jaipur-New Delhi.
5. Special Functions for Engineers and Mathematicians, L.C. Andrews, Macmillan Co., New York, 1985.

Integral University, Lucknow
Department of Mathematics

Pre Ph. D. course work

Subject: Hydrodynamic and Hydromagnetic Instability **Subject Code: MT606**
(w.e.f. session 2015-2016)

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UNIT-I Thermal Instability -I

Physical problem (Benard problem), basic hydrodynamical equations, initial stationary state solution, non linear perturbation equations, Principle of linearization, linear perturbation equations, normal mode analysis, principle of exchange of stabilities, exact solution when instability set in as stationary convection for two free boundaries.

UNIT-II Thermal Instability -II

Thermal instability in presence of magnetic field; Physical problem, governing equations, initial stationary state solution, non linear perturbation equations, Principle of linearization, linear perturbation equations, normal mode analysis, principle of exchange of stabilities, the case when instability set in as stationary convection for two free boundaries.

UNIT-III Instability of inviscid Homogeneous shear flow

Physical problem, governing equations, initial stationary state solution, perturbation equations, normal mode analysis, derivation of Rayleigh equation, mathematical eigen value problem, Rayleigh(1880), Fjortoft (1950), Banerjee – Shandil (1997) results on point of inflexion and Howard's (1961) semi circle theorem on complex wave velocity.

UNIT-IV Instability of inviscid Heterogeneous shear flow

Physical problem, governing equations, initial stationary state solution, perturbation equations, normal mode analysis, derivation of Taylor Goldstein equation, mathematical eigen value problem, different form of TG equation using transformations, Synge (1933), Miles(1961)results on phase velocity, Howard's (1961) semi circle theorem, Banerjee Shandil and Jain (2008) theorem on Howard semi circle.

UNIT-V Fundamentals of Mathematical Statistics

Probability distribution, Probability density function, Mean and standard deviation of a continuous random statistical variate, moment about mean, moment about any point, skewness and kurtosis, condition for symmetry of the distribution, relation between mean, standard deviation and range of variate; semi circle theorem, result on reduction of semi circle (Khan, *et. al*. 2009).

REFERENCES:

1. *Text Book of Fluid Dynamics by F. Charlon*
2. *Fluid Mechanics by Stgreeter and Wylie, M C Graw Hill*
3. *Introduction to fluid Mechanics by Fox and M c Donald.*
4. *Hydrodynamic and Hydromagnetic stability by S. Chandrashekhar, dover Publication, New York (1981)*
5. *Advanced Fluid Dynamics by Murlidhar and Biswas.*
6. *Hydrodynamic and Hydromagnetic stability by Banerjee, M.B. and Gupta, J.R. Silver Line publication, Shimla*
7. *Hydrodynamic stability by Drazin and Reid, Cambridge University Press.*
8. *Stability of Fluid Motion- II by D.D. Joseph, Springer- Verlag, New York.*
9. *The Thoery of Hydrodynamic stability by C.C. Lin, Cambridge University Press.*
10. *Mathematical Statistics by Gupta and Kapoor, Sultan Chand Pub. New Delhi.*
11. *An introduction to Magneto-Fluid Mechanics by V.C.A. Farraro and C. Plumpton, Oxford University Press (1996).*

Integral University, Lucknow
Department of Mathematics

Pre Ph. D. course work

Subject: Algebraic Structures **Subject Code: MT607**

(w.e.f. session 2017-2018)

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Unit 1

Quotient and Symmetric Groups: Normal subgroups, Quotient group, Homomorphism, Automorphism, Permutations, Symmetric groups, Cayley theorem

Unit 2

Structure Theory of Groups: Cauchy Theorem, Direct Products, Sylow theorem's, Finite Abelian Groups.

Unit 3

Solvable Groups and Jordan-Holder Theorem: Generators of a subgroup, Derived subgroups, Normal series, Solvable groups, Jordan Holder Theorem

Unit 4

Subdirect Sum of Rings: Subdirect Sum of Rings, Definitions and fundamental properties, Zorn's Lemma, Subdirectly irreducible rings, Boolean Rings

Unit 5

Prime ideals and prime radicals: Prime ideals and m-system, Semiprime Ideals, The prime radical of rings, Prime rings, The descending chain condition and The prime radicals.

Reference Books:

1. Topics in Algebra, I. N. Herstein, Wiley India Pvt. Limited.
2. Algebra: Michael Artin, Pearson Education.
3. Basic Abstract Algebra, P. B. Bhattacharya, S. K. Jain, S. R. Nagpaul, Cambridge University Press
4. Topics in Ring Theory, I. N. Herstein, University of Chicago Press.
5. The theory of Rings, N.H. McCoy, Chelsea Publishing Company, Bronx, NewYork.

Integral University, Lucknow

Department of Mathematics & Statistics

Pre PhD Course Work

Subject Name: Advanced Linear Algebra **Subject Code:** MT608

(w.e.f. session 2020-21)

UNIT-I

Vector spaces: subspaces, sums and direct sums; Finite dimensional vector spaces: bases and dimensions; Linear maps: null-spaces and range, invertibility.

UNIT-II

Eigen values and eigenvectors, Schur's theorem (real and complex versions), Spectral theorems for normal and Hermitian matrices (real and complex versions).

UNIT-III

Matrix norms, spectral radius formula, relationships between matrix norms, Singular value decomposition (SVD), polar decomposition, QR factorization.

UNIT-IV

General perturbation theory for matrices, Bauer-Fike and Henrici theorems, Hoffman-Wielandt theorem.

UNIT-V

Matrix pencils, eigenvalues and eigenvectors of regular pencils, triangular and Weierstrass forms, Kronecker canonical form.

Texts/References:

1. R. A. Horn and C. R. Johnson, Matrix Analysis, CUP, 1985.

2. G.W. Stewart and J. G. Sun, Matrix Perturbation Theory, Academic Press, 1990.

3. Vivek Sahai and Vikash Bist, Linear Algebra, Narosa Publishing House

4. Schaum's Outline, Linear Algebra, McGraw-Hill Publisher

5. G.H. Golub and C.F. Van Loan, Matrix Computations, Third ed. The Johns Hopkins University Press, Baltimore, MD, 1996.

Integral University, Lucknow

Department of Mathematics & Statistics

Pre PhD Course Work

Subject Name: Fuzzy Algebraic Structures **Subject Code:** MT609

(w.e.f. session 2020-21)

Unit 1:
Basic Concept: Crisp sets and fuzzy sets, Membership function, Operation on fuzzy sets, Equivalence relations, Semigroups, Groups, Semirings, Rings, Polygroups, Hypergroups and H_v-Structure, Codes.

Unit 2:
Fuzzy Semigroups: Fuzzy ideals of Semigroups, Fuzzy Regular and Intra-regular semigroups, Fuzzy Completely Regular and Weakly Completely Regular Semigroups, Fuzzy Congruences on a Semigroup, Prime Fuzzy Ideals.

Unit 3:
Fuzzy Semirings: Fuzzy ideals of semirings, Fuzzy subsemimodules over semirings, Fuzzy k-ideals of semirings, Fuzzy ideals with thresholds.

Unit 4:
Fuzzy Hyper-Structure: Fuzzy polygroups, Fuzzy H_v-Subgroups of H_v-Groups, H_v-Ideals, Interval-Valued Fuzzy H_v-Ideals, F-Hypergroups.

Unit 5:
Fuzzy Codes: Fuzzy codes, Prefix codes, Algebraic properties of fuzzy prefix codes on a free monoid, Fuzzy prefix codes related to Fuzzy Factor Theorems, An Algorithm of test for fuzzy codes, Code Theory and fuzzy Subsemigroups, Codes Over Languages.

Text Books:

1. J. N. Mordeson, D. S. Malik, N. Kuroki: Fuzzy Semigroups, Springer-Verlag Berlin Heidelberg 2003.
2. J. Ahsan, J. N. Mordeson, M. Shabir: Fuzzy Semirings with Applications to Automata Theory, Springer-Verlag Berlin Heidelberg 2012.
3. B. Davvaz,I. Cristea: Fuzzy AlgebraicHyperstructures-An Introduction, SpringerInternational Publishing Switzerland 2015.
4. J. M. Howie: Fundamental of Semigroups, Oxford University Press 1995.

Integral University, Lucknow

Department of Mathematics & Statistics

Pre PhD Course Work

Subject Name: Theory of Approximation **Subject Code: MT610**

(w.e.f. session 2020-21)

Unit-1.
Concept of best approximation in a normed linear space, Existence of best approximation, Uniqueness problem, Uniform convexity, Strict convexity, continuity of best approximation operator.

Unit-2.
Convergence of Fourier series, Divergence of Fourier, Weierstrass approximation theorem, Bernstein polynomials, Weierstrass second theorem, Monotone operators, Modulus of continuity and its properties, Lipschitz class and its properties, Banach fixed point theorem, Jackson's theorems,

Unit-3.
General linear families, characterization theorem, Haar conditions, Alteration theorem, Abel and matrix summability, Abel transformation, Product summability transform.

Unit-4.
Strong unicily theorem, Haar's theorem, the convergence of Jackson theorems, Bernstein inequality, Bernstein theorems, Zygmund theorem, Summability theorems.

Unit-5.
Positive linear operators with their approximation properties, Local and Global approximation, Rate of convergence, weighted approximation, approximation in quantum calculus and (p, q)-calculus.

BOOKS:

1. P.P. Korovkin Edited by A.R. Gairola, Korovkin's, Linear operators and approximation theory, Ist revised edition**,** Hindustan Publishing Corporation, New Delhi, India, 2017.
2. G.G. Lorentz, Bernstein Polynomials, University of Toronto Press, Toronto, 1997.
3. I.P. Natanson, Constructive Function theory Vol. I, Uniform approximation, Frederick Ungar, New York, 1964.
4. M.J.D. Powell, Approximation theory and methods, Cambridge University Press, 1981.
5. H.N. Bhaskar& D.V. Pai, Fundamental of approximation theory, Narosa Publishing House.
6. Chandrasekhar Rao, "Functional Analysis", Alpha Science International Ltd, United Kingdom.
7. R.R. Goldberg, Methods in Real Analysis, John Wiley & Sons, Inc.1976.
8. George Bachman, Lawrence Narici & Edward Beckenstein, Fourier and Wavelet analysis, Springer2002.
9. Victor Kac, Quantum Calculus, Springer.